11: What is $\frac{1}{3}$ of $\frac{1}{3}$?

We now know that $\frac{1}{3}$ of (a) $\frac{1}{3}$ is $\frac{1}{3}$ x $\frac{1}{3}$ ('of' = 'times')

The rule for multiplying fractions is:

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$$

which means that $\frac{1}{3}\text{x}\frac{1}{3}=\frac{1\times 1}{3\times 3}=\frac{1}{9}$. But where does this rule come from?

Let us apply this rule using the example of the cake:

Think of a piece that is one third ($\frac{1}{3}$) of the cake, i.e. which fits into the whole cake 3 times. Now visualize a slice which is one third of the above piece. Clearly this will fit 9 times into the whole cake, i.e. it is $\frac{1}{9}$ of the cake. This slice is $\frac{1}{3}$ of $\frac{1}{3}$, the same as $\frac{1}{3}\text{x}\frac{1}{3}$.

So we have established why $\frac{1}{3}\text{x}\frac{1}{3}=\frac{1}{9}$ which is the same as $\left(\frac{1\times 1}{3\times 3}\right)$, as in the rule given above.

Had we started with a *fifth* ($\frac{1}{5}$)of the cake, and then taken a slice one third of that fifth, clearly 15 such slices would have fitted into the whole cake, i.e. this slice would be one 15th of the cake (which we can write as $\frac{1}{3\times 5}$, because the 15, of course, came from the 3 x 5.

So we have established that $\frac{1}{3}\text{x}\frac{1}{5}=\frac{1(\times 1)}{3\times 5}=\frac{1}{15}$ (again as in the rule).

Now, had we started with $\frac{1}{5}$ of *seven* cakes (rather than of 1), i.e. $\frac{7}{5}$, and then went on, as before, taking one third of the this, then the final piece would have been 7 times bigger (than the above $\frac{1}{3\times 5}$), namely $\frac{1(\times 7)}{3\times 5}$.

Finally, for the 'sub-division' of the cake, had we taken two thirds ($\frac{2}{3}$), rather than one third, the result would be yet another 2 times bigger, i.e. $\frac{2\times 7}{3\times 5}$.

So, we have shown why $\frac{2}{3}\text{x}\frac{7}{5}=\frac{2\times 7}{3\times 5}$, thus showing where the general rule

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$$ comes from.