22: How to write $(A+B)^2$ without the ( )

The explanation consists of two short and simple stages:

(i) 3 cats + 4 cats = 7 cats.

The 7 came from (3+4), so we can write:

3 cats + 4 cats = (3 + 4) cats

We don't need the whole cat, 'C' will do:

3 C + 4 C = (3+4) C (3 C is the same as $3\times$C)

This works just as well with any pair of numbers, say, 'n' and 'p' (instead of the 3 & 4), so:

n $\times$ C + p $\times$ C = (n+p) $\times$ C

Reversing the order, and using different letters:

$$C\times(a+b)=C\times a+C\times b$$

The '$\times$' are sometimes omitted or replaced by a dot, i.e.

$$C(a+b)=C\cdot a+C\cdot b$$

Always remember that $C(a+b)$ is the same as $(a+b)C$. This is one of the most common and fundamental tools in Algebra, and now we know where it comes from.

(ii) K^2 is shorthand for $K \times K$. This notation works with anything, even with 🐈, namely

🐈 ^{2} = 🐈 x 🐈

And what is good for 🐈, is good for (A+B):

$$(A+B)^2=(A+B)\times(A+B).$$

How do we rewrite $(A+B)\times(A+B)$ without the ( )'s? We know what to do if, in place of the 1st $(A+B)$ we had a single symbol, like the *C* in stage (i), so let us replace, *temporarily*, the 1st $(A+B)$ with some single symbol, 🐈 say:

🐈 x $(A+B)$, which we know is 🐈 x A + 🐈 x B

Now, return the 1st $(A+B)$ for the 🐈:

$$(A + B) \text{ x } A + (A + B) \text{ x } B$$

Here we now have two of these exercises as in (i): A x A + B x A + A x B + B x B

This can be written as:

$$A^2 + 2AB + B2$$

So,

$$(A+B)^2 = A^2 + 2AB + B^2 \text{ (all thanks to the 🐈)}$$