13:   900 000 ÷ 300 = ?   30 ÷ 100 = ?

Write $900{,}000 \div 300$ as $\dfrac{900000}{300}$.   We are used to seeing the divided – often called the numerator - above the fraction line and the divider (or denominator) below the fraction line. We can write 900 000 as 9 x 100 000 and 300 as 3 x 100.

The task then looks like $\dfrac{9 \times 100000}{3 \times 100}$.

Let us first consider $\dfrac{9}{3}$, which equals 3.

We should have divided something 100 000 time bigger than 9, making the result 100 000 times bigger. We should also have divided by something 100 times bigger than 3, making the result 100 times smaller. What results from making something 100 000 times bigger and then 100 times smaller? You can think of the latter as 10 times smaller and then 10 times smaller again, each time knocking one 0 off the 100 000, leaving 1000 as the net 'adjustment' to the 3.

So $\dfrac{900000}{300} = 3000$

In the same way, $\dfrac{9 \times m}{3 \times m}$ is just $\dfrac{9}{3}$ because there is the same m -fold increase as the m-fold decrease.

Note that this does *not* work with, for example, $\dfrac{9 + m}{3 + m}$ (Try taking m = 3 which gives $\dfrac{12}{6}$ which is 2, not 3).

In the case of 30 ÷ 100, (i.e. 30 %), again we start with $\dfrac{3}{1}$ and then make it ten times bigger (because the divided is 30, not 3) and one hundred times smaller (because the divider is 100, not 1). The net adjustment is 10 times smaller than the 3, namely 0.3

Remember: The correct value of a fraction can also always be found by long division.

Try working out as decimals what these are:

$$\frac{1}{7}, \frac{2}{7} \cdots \frac{6}{7}$$

What happens, and why didn't we ask for $\dfrac{7}{7}$?